UGVA-DPT 1995/05-890 ; BARI-TH/95-200

STRONG COUPLING OF EXCITED HEAVY MESONS*

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(May 1995)

Abstract

We compute the strong coupling constant $G_{B^{**}B\pi}$ ($G_{D^{**}D\pi}$), where B^{**} (D^{**}) is the 0^+ $P-$wave $b\bar{q}$ ($c\bar{q}$) state, by QCD sum rules and by light-cone sum rules. The two methods give compatible results in the limit $m_Q \to \infty$, with a rather large value of the coupling constant. We apply the results to the calculation of the hadronic widths of the positive parity B and D states and to the chiral loop contribution to the ratio f_{D_s}/f_D.

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*Partially supported by the Swiss National Foundation

I. INTRODUCTION

In the light quark ($q = u, d, s$) zero mass limit ($m_q \to 0$) and in the heavy quark ($Q = c, b$) infinite mass limit ($m_Q \to \infty$) Quantum Chromodynamics exhibits symmetries that are not present in the finite mass theory: chiral $SU(3)_L \times SU(3)_R$ symmetry, heavy quark spin and flavour symmetries [1], as well as the velocity superselection rule [2], valid insofar only strong interactions are considered.

All these symmetries can be used to build up an effective chiral lagrangian [3] for light pseudoscalar mesons and heavy ($Q\bar{q}$) negative parity ($J^P = 0^-, 1^-$) mesons. This effective lagrangian contains a symmetric term plus corrections to the heavy-light symmetries such as, for example, terms proportional to powers of $1/m_Q$ or terms proportional to powers of the light quark masses m_q. In the spirit of the chiral effective theory, the resulting lagrangian is also an expansion in the light meson fields derivatives.

Besides the octet of the pseudo-Goldstone bosons π, K, η and the (D, D^*), (B, B^*) states, one can include the light vector mesons belonging to the low lying $SU(3)$ nonet: ρ, K^*, ω_8, ω_0, using the so-called hidden symmetry approach [4,5]. As well known, in this approach [6] the lagrangian exhibits an extra local gauge symmetry $SU(3)_H$, and the 1^- light meson octet represents its gauge bosons. They acquire a mass because $SU(3)_H$ is spontaneously broken. Quite recently it has been observed [7] that, if $SU(3)_H$ is unbroken, a new symmetry (vector symmetry) arises. Its implications for the heavy-light chiral lagrangian have been examined in Ref. [8]. Additional notions arise, in general, when also axial-vector bosons are present [9].

Another extension of the heavy-light lagrangian is obtained by including effective fields describing positive parity ($Q\bar{q}$) mesons. According to the value of the angular momentum of the light degrees of freedom, ($s_\ell^P = \frac{1}{2}^+$, $\frac{3}{2}^+$) the Heavy Quark Effective Theory [10,11] predicts the existence of two multiplets, the first one comprising 0^+ and 1^+ mesons, the second one containing 1^+, 2^+ states. The role of the $\frac{1}{2}^+$ doublet (0^+, 1^+) in some applications of

chiral perturbation theory has been considered in [12][1]. Another application is in the realm of the semileptonic D and B decays [4].

The aim of the present paper is to give an estimate of the strong coupling constant describing the interaction of the pseudoscalar light mesons with the positive parity ($s_\ell^P = \frac{1}{2}^+$) and negative parity ($s_\ell^P = \frac{1}{2}^-$) heavy mesons and to make quantitative estimates of the effect of the positive parity heavy mesons in some calculations in chiral perturbation theory.

After a review of the heavy-light chiral lagrangian in Section II, we consider two sum rules for these coupling constants: the first one, based on the method of the single Borel transform (in the soft pion limit), is discussed in Section III, while in Section IV we derive these couplings by the method of the light-cone sum rules [13] (for a review see [14]).

In Section V we compute the decay widths of the excited positive parity heavy mesons, and we estimate the mixing angle between $\frac{1}{2}^+$ and $\frac{3}{2}^+$ axial vector states.

In Section VI we comment on the role of the positive parity heavy mesons in the chiral loop contributions to the ratio f_{D_s}/f_D, which, as observed in [12], may be considerable. We find the numerical result:

$$\frac{f_{D_s}}{f_D} = 1.09 \tag{1.1}$$

where part of the SU(3) violation effect $f_{D_s}/f_D \neq 1$ may be attributed to positive parity states.

Finally, in Section VII we draw our conclusions.

II. THE HEAVY-LIGHT CHIRAL LAGRANGIAN

In the effective heavy-light chiral lagrangian, the ground state ($s_\ell^P = \frac{1}{2}^-$) heavy mesons are described by the 4×4 Dirac matrix

[1]There is no coupling of the ($s_\ell^P = \frac{3}{2}^+$) states to the $(0^-,\ 1^-)$ heavy meson doublet and to the pseudoscalar Goldstone bosons at the lowest order in the chiral expansion [11].

$$H_a = \frac{(1+\not v)}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5] \tag{2.1}$$

where v is the heavy meson velocity, $P_a^{*\mu}$ and P_a are annihilation operators of the 1^- and 0^- $Q\bar{q}_a$ mesons ($a = 1, 2, 3$ for u, d and s): for charm, they are D^* and D respectively. Similarly, the positive parity 1^+ and 0^+ ($s_\ell^P = \frac{1}{2}^+$) are described by

$$S_a = \frac{1+\not v}{2}\left[D_1^\mu \gamma_\mu \gamma_5 - D_0\right] \ . \tag{2.2}$$

It should be observed that all the operators appearing in Eq. (2.1) and Eq. (2.2) have dimension $\frac{3}{2}$ since they contain a factor $\sqrt{m_P}$ in their definition.

As for the octet of the pseudo Goldstone bosons, one uses the exponential form:

$$\xi = \exp\frac{iM}{f_\pi} \tag{2.3}$$

where

$$M = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \tag{2.4}$$

and $f_\pi = 132\ MeV$.

The lagrangian describing the fields H, S and ξ and their interactions, under the hypothesis of chiral and spin-flavour symmetry and at the lowest order in light mesons derivatives is:

$$\begin{aligned} \mathcal{L} &= \frac{f_\pi^2}{8} < \partial^\mu\Sigma\partial_\mu\Sigma^\dagger > + i < H_b v^\mu D_{\mu ba}\bar{H}_a > \\ &+ < S_b\,(i\,v^\mu D_{\mu ba}\ -\ \delta_{ba}\,\Delta)\bar{S}_a > + \,i\,g < H_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\bar{H}_a > \\ &+ i\,g' < S_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\bar{S}_a > + [i\,h < S_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\bar{H}_a > \ + h.c.] \end{aligned} \tag{2.5}$$

where $< \ldots >$ means the trace, and

$$D_{\mu ba} = \delta_{ba}\partial_\mu + \mathcal{V}_{\mu ba} = \delta_{ba}\partial_\mu + \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)_{ba} \tag{2.6}$$

$$\mathcal{A}_{\mu ba} = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)_{ba} \ ; \tag{2.7}$$

$\Sigma = \xi^2$ and Δ is the mass splitting of the S_a states from the ground state H_a. Numerically we use $\Delta = 500 \pm 100$ MeV, an estimate based on quark model [15] and QCD sum rules [16] computations of the masses of the excited $s_\ell^P = \frac{1}{2}^+$ mesons.

In Ref. [17] a QCD sum rule has been considered to compute the strong coupling constant g appearing in Eq.(2.5), with the result (at the order $\alpha_s = 0$):

$$g = 0.39 \pm 0.16 \; . \tag{2.8}$$

This result, valid in the soft pion limit, has been confirmed by a subsequent, independent analysis, based on the method of light-cone sum rules [18]:

$$g = 0.32 \pm 0.02 \; . \tag{2.9}$$

The results of similar sum rules for the strong coupling constant h in Eq.(2.5) will be presented in the subsequent Sections.

III. QCD SUM RULE FOR h

Let use define the strong amplitude

$$G_{B^{**}B\pi} \; = < \pi^+(q) \; B^o(q_2) | B^{**+}(q_1) > \tag{3.1}$$

where B^{**} is the 0^+ state in the $s_\ell^P = \frac{1}{2}^+$ doublet. The amplitude $G_{B^{**}B\pi}$ is related to the strong coupling constant h appearing in the heavy-light chiral lagrangian (2.5) by the formula:

$$G_{B^{**}B\pi} \; = - \sqrt{m_B m_{B^{**}}} \; \frac{m^2_{B^{**}} - m^2_B}{m_{B^{**}}} \; \frac{h}{f_\pi} \; . \tag{3.2}$$

In the limit $m_b \to \infty$ one has:

$$\begin{aligned} m_B &= m_b + \omega + \mathcal{O}\left(\frac{1}{m_b}\right) \\ m_{B^{**}} - m_B &= \Delta \; + \; \mathcal{O}\left(\frac{1}{m_b}\right) \end{aligned} \tag{3.3}$$

and

$$G_{B^{**}B\pi} \simeq -\frac{2h}{f_\pi}\, m_b\, \Delta \tag{3.4}$$

In order to derive a sum rule for $G_{B^{**}B\pi}$ and h, we consider the correlator:

$$A_\mu = i \int dx < \pi^+(q)|T(j_5(x)V_\mu(0)|0 > e^{-iq_2 x} = A(q_1^2, q_2^2, q^2) q_\mu + B(q_1^2, q_2^2, q^2) P_\mu \tag{3.5}$$

where $j_5 = \overline{u} i \gamma_5 b$, $V_\mu = \overline{b}\gamma_\mu d$, $q = q_1 - q_2$ and $P = q_1 + q_2$. The scalar functions A and B satisfy dispersion relations (D.R.) that are computed, according to the method of QCD sum rules, in two ways: either by saturating the dispersion relation by physical hadronic states, or by means of the operator product expansion (O.P.E.). In Ref [17] we considered the dispersion relation for the scalar function A and we used it to compute the coupling constant $G_{B^*B\pi}$ defined by the matrix element:

$$< \pi^+(q)\; B^o(q_2)|B^{*+}(q_1, \epsilon) > = \; G_{B^*B\pi}\; \epsilon_\mu \cdot q^\mu \; . \tag{3.6}$$

We obtained the result:

$$f_B\; f_{B^*}\; G_{B^*B\pi} = 0.56 \pm 0.12\; GeV^2 \tag{3.7}$$

while for charm we obtained:

$$f_D\; f_{D^*}\; G_{D^*D\pi} = 0.34 \pm 0.08\; GeV^2 \; . \tag{3.8}$$

The leptonic decay constants appearing in the previous equations are defined as follows:

$$\begin{aligned} < 0|\overline{b}\gamma^\mu \gamma_5 d|B(p) > \; &= i f_B p^\mu \\ < 0|\overline{u}\gamma^\mu b|B^*(p, \epsilon) > \; &= i\; m_{B^*} f_{B^*}\; \epsilon^\mu \; . \end{aligned} \tag{3.9}$$

It should be noticed that the phases in the previous equation are consistent with the definition of the weak current in the effective theory, see below Eq.(6.3).

In the following we shall make use also of the matrix element:

$$< 0|\overline{u}\gamma^\mu b|B^{**}(p) >= i f_{B^{**}}\; p^\mu \; . \tag{3.10}$$

It may be observed that the hadronic side of the sum rule for A also includes the B^{**} pole in the variable q_1^2 (the low lying pole in the variable q_2^2 is provided, of course, by the B meson). In [17] one gets rid of it by exploiting the tiny mass difference between B and B^*, which allows to include the $J^P = 0^+$ pole in the so called *parasitic terms* that may be appropriately parametrized. Here we are interested precisely in this pole and, in order to obtain it, we consider the dispersion relation for the scalar function B in Eq.(3.5) that should be written in general as follows [19] :

$$
\begin{aligned}
B(q_1^2, q_2^2, q^2) = \frac{1}{\pi^2} \int ds ds' \frac{\rho(s, s', q^2)}{(s - q_1^2)(s' - q_2^2)} + \\
+ P_1(q_1^2) \int ds' \frac{\rho_1(s', q^2)}{s' - q_2^2} + P_2(q_2^2) \int ds \frac{\rho_2(s, q^2)}{s - q_1^2} + P_3(q_1^2, q_2^2, q^2) \; .
\end{aligned}
\tag{3.11}
$$

As proven in [20] $P_1(q_1^2) = P_2(q_2^2) = 0$ since only this value of the polynomials $P_1(q_1^2)$ and $P_2(q_2^2)$ is compatible with the vanishing of the form factors for large values of their arguments, as predicted by quark counting rules. The subtraction polynomial $P_3(q_1^2, q_2^2)$ does not contribute to the sum rule since it vanishes after the Borel transform; therefore we shall neglect it in the sequel.

In order to compute the D.R. for B, in this Section we make the approximation of the soft pion limit (S.P.L.): $q \to 0$. This approximation presents the advantage of a considerable simplification of the calculations; moreover in this scheme the limit $m_Q \to \infty$ can be performed in a well defined way. On the other hand it might be argued that in the decay $B^{**} \to B\pi$ the pion momentum is not constrained to be small, since $m_{B^{**}} - m_B \simeq \Delta \simeq 500$ MeV. We shall comment on the uncertainties introduced by a small pion momentum approximation in the next Section, where we shall present a light-cone sum rules calculation of $G_{B^{**}B\pi}$ which is not based on S.P.L.

The problem arising in the soft pion limit is related to the fact that $q = 0$ implies $q_1^2 = q_2^2$. As a consequence, one cannot perform a double Borel transform in the variables q_1^2 , q_2^2, and the single Borelization procedure has to be used. As well known [19] in this way one introduces unwanted not-exponentially suppressed contributions (the so called *parasitic*

terms) that have to be estimated[2]. We shall show in the sequel how this problem can be solved. For the time being we consider the result of computing $B(q_1^2, q_1^2, 0)$ by OPE in the soft pion limit. The result of a straightforward analysis (much similar to that considered in Refs. [17,21]) is as follows:

$$B(q_1^2, q_1^2, 0) = B^{(0)} + B^{(1)} + B^{(2)} + B^{(3)} + B^{(4)} + B^{(5)} \tag{3.12}$$

where

$$\begin{aligned}
B^{(0)} &= -\frac{<\overline{u}u>}{f_\pi}\frac{1}{q_1^2 - m_b^2} \\
B^{(1)} &= 0 \\
B^{(2)} &= -\frac{m_0^2 <\overline{u}u>}{4f_\pi(q_1^2 - m_b^2)^2}\left[1 - \frac{2m_b^2}{q_1^2 - m_b^2}\right] \\
B^{(3)} &= 0 \\
B^{(4)} &= \frac{m_0^2 <\overline{u}u>}{4f_\pi(q_1^2 - m_b^2)^2} \\
B^{(5)} &= 0\ .
\end{aligned} \tag{3.13}$$

In Eqs. (3.13) $<\overline{u}u>$ is the quark condensate ($<\overline{u}u>= -(240MeV)^3$), m_0^2 is defined by the equation

$$<\overline{u}g_s\sigma\cdot Gu>= m_0^2 <\overline{u}u> \tag{3.14}$$

and its numerical values is: $m_0^2 = 0.8\ GeV^2$. The origin of the different terms in Eq.(3.12) is as follows. $B^{(0)}$ is the leading term in the short distance expansion; $B^{(1)}$, $B^{(2)}$ and $B^{(3)}$ arise from the expansion of $j_5(x)$ at the first, second and third order in powers of x; $B^{(4)}$ and $B^{(5)}$ arise from the expansion of the heavy quark propagator at the second order and from the zeroth and first term in the expansion of $j_5(x)$ respectively. We have considered all the operators with dimension $D \leq 5$ in the O.P.E. of the currents appearing in Eq.(3.5).

Let us now compute the hadronic side of the sum rule, that we call B_{had}. We divide the integration region into two regions D_1, D_2, as depicted in Fig. 1. D_1 is bounded by the lines $s = m_b^2, s' = m_b^2$ and $s + s' = C$. We assume that C satisfies the bounds:

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[2]A similar situation is met computing $g_{B^*B\pi}$ by this method [17].

$$min(s'_0 + m^2_{B^{**}}, s_0 + m^2_B) \geq C \geq m^2_{B^{**}} + m^2_B \tag{3.15}$$

where s_0 and s'_0 are thresholds for continuum production in the variables s and s' respectively. The bound in (3.15) is chosen in such a way that, inside D_1, only the poles B^* and B^{**} (in the variable s) and B (in the variable s') are included. Their contribution is as follows:

$$B_{pole}(q_1^2, q_1^2, 0) = -\frac{G_{B^*B\pi} f_B f_{B^*} m_B (m^2_{B^*} - m^2_B)}{4 m_b m_{B^*} (q_1^2 - m^2_{B^*})(q_1^2 - m^2_B)} - \frac{G_{B^{**}B\pi} f_B f_{B^{**}} m^2_B}{2 m_b (q_1^2 - m^2_{B^{**}})(q_1^2 - m^2_B)} \ . \tag{3.16}$$

We observe that, due to the presence of the factor $m^2_{B^*} - m^2_B$ the contribution of the 1^- pole is strongly depressed as compared to the term describing the 0^+ resonance; in the $m_b \to \infty$ limit, the ratio of the two terms is $O(\frac{1}{m_b^2})$. We have checked that this suppression holds not only for the beauty, but also for the charm, where the vector meson pole contribution is less than 3% of the 0^+ pole. For this reason we shall omit the 1^- pole in the sequel and we shall take only the scalar contribution in Eq.(3.16).

Let us now consider the region D_2, where a continuum of resonances contributes. Following Ref. [22] we assume the following model for the hadronic dispersive function in this region:

$$\rho(s, s') \ = \ f(s, s') \delta(s - s') \theta(s \ + \ s' \ - \ C) \qquad\qquad (region \ \ D_2) \ . \tag{3.17}$$

In order to justify this choice let us introduce the variables $z = s - s'$ and $y = \frac{s+s'}{2}$. In these new variables we write the hadronic side of the sum rule as follows:

$$B_{had}(q_1^2, q_1^2, 0) = B_{pole}(q_1^2, q_1^2, 0) + B_{cont}(q_1^2) \tag{3.18}$$

where the contribution of the continuum of resonances (region D_2) is as follows:

$$B_{cont}(q_1^2) = \frac{1}{\pi^2} \int_{C/2}^{\infty} dy \int_{-2(y - m_b^2)}^{+2(y - m_b^2)} dz \frac{1}{(y + \frac{z}{2} - q_1^2)(y - \frac{z}{2} - q_1^2)} \rho(y + \frac{z}{2}, y - \frac{z}{2}) \ . \tag{3.19}$$

Let us now rewrite the leading term in the O.P.E. expansion as an integral over the variables $z, \ y$:

$$B^{(0)} = -\frac{<\overline{u}u>}{f_\pi}\frac{1}{q_1^2 - m_b^2} =$$

$$= \frac{<\overline{u}u>}{f_\pi}\int_{m_b^2}^{\infty} dy \frac{1}{(y - q_1^2)^2}\int_{-2(y-m_b^2)}^{+2(y-m_b^2)} dz\delta(z) \ . \tag{3.20}$$

To relate (3.19) and (3.20), one imposes local duality in y [22]; this means that inside the region D_2, i.e. for $y \geq C/2$, the arguments of the y-integrals in Eqs. (3.19) and (3.20) should be equal. This implies that (3.17) is valid with

$$f(s,s') = \pi^2\frac{<\overline{u}u>}{f_\pi} \tag{3.21}$$

which fixes the model of the hadronic continuum in the region D_2. A justification for local duality in the variable y can be obtained by explicit calculations in particular models [24].

Using these results we obtain, after the Borel transform, the following sum rule, which as we have already observed, is valid only in the soft pion limit:

$$\frac{G_{B^{**}B\pi} f_B f_{B^{**}} m_B^2}{m_{B^{**}}^2 - m_B^2}\left[e^{-m_B^2/M^2} - e^{-m_{B^{**}}^2/M^2}\right]$$
$$= -\frac{2m_b <\overline{u}u>}{f_\pi}\left[e^{-m_b^2/M^2}\left[1 - \frac{m_b^2 m_0^2}{4M^4}\right] - e^{C/(2M^2)}\right] \ . \tag{3.22}$$

The sum rule is valid provided one imposes two conditions on the Borel parameter M^2. First one requires that the contribution of the continuum does not exceede the pole contributions (for larger values this would produce uncontrollable uncertainties in $G_{B^{**}B\pi}$ due to our poor knowledge of the continuum); this fixes the upper bound for M^2. On the other hand, for the O.P.E to be meaningful, we have to impose that the higher power terms in $1/M^2$ in Eq.(3.22) have decreasing values, which fixes the lower bound for M^2. For the beauty sector we use the values $m_{B^{**}} - m_B = \Delta \approx 500 \pm 100$ MeV, $m_b = 4.6\ GeV$, the parameter C in the range $(61 - 64)\ GeV^2$ (with thresholds $s_0 = {s'}_0 \simeq 36\ GeV^2$). The previous criteria are satisfied for M^2 in the range $(15 - 25)\ GeV^2$. We get the result:

$$G_{B^{**}B\pi} f_B f_{B^{**}} = 0.43 \pm 0.06\ GeV^3 \ . \tag{3.23}$$

For charm, we use $m_{D^{**}} - m_D \approx \Delta = 500 \pm 100$ MeV, $m_c = 1.35\ GeV$, and the parameter C in the range$(10-14)\ GeV^2$ (with thresholds $s_0 = s'_0 \simeq 6\ GeV^2$). The criteria for M^2 are satisfied in the range $M^2 = (2-8)$ GeV^2, and one gets the result:

$$G_{D^{**}D\pi} f_D f_{D^{**}} = 0.38 \pm 0.11\ GeV^3\ . \tag{3.24}$$

To get $G_{B^{**}B\pi}$ and $G_{D^{**}D\pi}$ we use $f_B = 0.18 \pm 0.03\ GeV$ and $f_{B^{**}} = 0.18 \pm 0.03\ GeV$ [16] obtaining:

$$G_{B^{**}B\pi} = 13.3 \pm 4.8\ GeV\ , \tag{3.25}$$

while, using $f_D = 0.195 \pm 0.020\ GeV$ and $f_{D^{**}} = 0.17 \pm 0.02\ GeV$, [16] we obtain:

$$G_{D^{**}D\pi} = 11.5 \pm 4.0\ GeV\ . \tag{3.26}$$

We observe substantial violations of the scaling law $G_{D^{**}D\pi}/G_{B^{**}B\pi} \approx \frac{m_c}{m_b}$. On the other hand the ratio $R = \frac{G_{B^{**}B\pi} f_B f_{B^{**}}}{G_{D^{**}D\pi} f_D f_{D^{**}}}$ is less sensitive to scaling violations (numerically we find $R \simeq 1.13$, to be compared to the scaling prediction $R = 1$).

Let us now take the limit $m_b \to \infty$. This limit is defined by Eqs. (3.3) and by a rescaling of the Borel parameter M^2:

$$M^2 \ = \ 2 m_b\, E\ . \tag{3.27}$$

In this way we obtain the asymptotic rule:

$$h \hat{F} \hat{F}^+ \left[1 \ - \ e^{-\Delta/E}\right] = 4 < \overline{u} u > \left[e^{\omega/E}(1 \ - \ \frac{m_0^2}{16E^2}) \ - \ e^{-\delta/E}\right]\ ; \tag{3.28}$$

the quantities δ, $\hat{F}$ and $\hat{F}^+$, defined by:

$$\begin{aligned} \delta &= \frac{1}{2m_b}\left[\frac{C}{2} - m_B^2\right] \\ \hat{F} &= f_B \sqrt{m_B} \\ \hat{F}^+ &= f_{B^{**}} \sqrt{m_{B^{**}}} \end{aligned} \tag{3.29}$$

remain finite in the infinite heavy quark mass limit, modulo logarithmic corrections. The constraints on the Borel parameter imply that E must be in the range $(0.5-1.4)\ GeV$.

Using $\omega = 0.50 \pm 0.07$ GeV, $\Delta = 500$ MeV, and $\delta = 400 \pm 50$ MeV the numerical outcome of the sum rule is:

$$h\hat{F}\hat{F}^{+} \; = \; -\,0.072 \pm 0.008 \, GeV^3 \tag{3.30}$$

(the results are weakly dependent on δ). By the values $\hat{F} = 0.30 \pm 0.05$ GeV$^{3/2}$ [23] and $\hat{F}^{+} \; = \; 0.46 \pm 0.06$ GeV$^{3/2}$ [16] we obtain:

$$h = \,-\,0.52 \pm 0.17 \; . \tag{3.31}$$

IV. A LIGHT-CONE SUM RULES CALCULATION OF h

An independent calculation of the strong coupling h can be carried out using a method that allows us to consider arbitrary momenta of the pion. We shall consider the correlation function

$$A(q_1^2, q_2^2, q^2) = i \int dx < \pi^{+}(q)|T(j_5(x) j(0)|0 > e^{-iq_2 x} \tag{4.1}$$

where $j_5 = i\overline{u}\gamma_5 d$ (as before) and $j = \overline{b}d$.

The method consists in expanding the T-product of the quark currents, appearing in Eq.(4.1), near the light-cone, in terms of non-local operators whose matrix elements can be written as wave functions of increasing twist.

This approach finds its origin in the analysis of hard exclusive processes in QCD [13]; within this framework, strong couplings ($g_{\omega\rho\pi}$, $g_{\pi NN}$), form factors (such as $\pi A \gamma^*$, the pion form factor at intermediate momentum transferred) and nucleon magnetic moments have been calculated [14]. Light-cone sum rules have also been used to calculate the form factors governing B and D meson semileptonic decays [25], and the radiative $B \to K^* \gamma$ transition [26].

In our calculation of h we follow the notations adopted by the recent paper by Belyaev et al. [18] devoted to the calculation of the coupling constants $g_{B^* B\pi}$ and $g_{D^* D\pi}$. Within the light-cone sum rules approach the correlator Eq.(4.1) can be written as follows:

$$A^{QCD}(q_1^2,q_2^2,q^2) = -\frac{2m_b < \bar{u}u >}{f_\pi}\int_0^1 du \frac{\varphi_P(u)}{m_b^2-(q_2+uq)^2} +$$
$$+ f_\pi(q_2\cdot q)\int_0^1 du\Big\{\frac{\varphi_\pi(u)}{m_b^2-(q_2+uq)^2} - \frac{4g_1(u)}{[m_b^2-(q_2+uq)^2]^2}\Big(1+\frac{2m_b^2}{m_b^2-(q_2+uq)^2}\Big)\Big\}$$
$$+ f_\pi(q_2\cdot q)\int_0^1 du\int \mathcal{D}\alpha_i \frac{(1-2u)\chi(\alpha_i)+\tilde{\chi}(\alpha_i)}{[m_b^2-(q_2+q(\alpha_1+u\alpha_3))^2]^2}\ . \tag{4.2}$$

The pion wave functions $\varphi_P(u)$, $\varphi_\pi(u)$ and $g_1(u)$ appear in the matrix elements of non-local quark operators [18]:

$$< \pi(q)|\bar{d}(x)\gamma_\mu\gamma_5 u(0)|0 > = -if_\pi q_\mu \int_0^1 du e^{iuqx}(\varphi_\pi(u)+x^2 g_1(u)+\mathcal{O}(x^4))$$
$$+ f_\pi(x_\mu - \frac{x^2 q_\mu}{qx})\int_0^1 du e^{iuqx} g_2(u)\ , \tag{4.3}$$
$$< \pi(q)|\bar{d}(x)i\gamma_5 u(0)|0 > = \frac{f_\pi m_\pi^2}{m_u+m_d}\int_0^1 du e^{iuqx}\varphi_P(u)\ , \tag{4.4}$$

where $\frac{f_\pi m_\pi^2}{m_u+m_d}$ is related to the light-quark condensate $< \bar{u}u >$ by the current algebra relation $\frac{f_\pi m_\pi^2}{m_u+m_d} = -2\frac{<\bar{u}u>}{f_\pi}$. It should be noticed that the path-ordered gauge factor $P\exp(ig_s\int_0^1 du x^\mu A_\mu(ux))$ has been omitted in the above matrix elements due to the choice of the light-cone gauge $x^\mu A_\mu(x)=0$.

The functions $\chi(\alpha_1,\alpha_2,\alpha_3)$ and $\tilde{\chi}(\alpha_1,\alpha_2,\alpha_3)$ are combinations of twist-four wave functions $\varphi(\alpha_1,\alpha_2,\alpha_3)$ and $\tilde{\varphi}(\alpha_1,\alpha_2,\alpha_3)$ [27]:

$$\chi(\alpha_1,\alpha_2,\alpha_3) = 2\varphi_\perp(\alpha_1,\alpha_2,\alpha_3) - \varphi_\parallel(\alpha_1,\alpha_2,\alpha_3) \tag{4.5}$$

and

$$\tilde{\chi}(\alpha_1,\alpha_2,\alpha_3) = 2\tilde{\varphi}_\perp(\alpha_1,\alpha_2,\alpha_3) - \tilde{\varphi}_\parallel(\alpha_1,\alpha_2,\alpha_3)\ , \tag{4.6}$$

where the functions φ and $\tilde{\varphi}$ parametrize the matrix elements of quark-gluon operators:

$$< \pi(q)|\bar{d}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(ux)u(0)|0 >=$$
$$f_\pi\Big[q_\beta\Big(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{q\cdot x}\Big) - q_\alpha\Big(g_{\beta\mu}-\frac{x_\beta q_\mu}{q\cdot x}\Big)\Big]\int \mathcal{D}\alpha_i\varphi_\perp(\alpha_i)e^{iqx(\alpha_1+u\alpha_3)}$$
$$+f_\pi\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i\varphi_\parallel(\alpha_i)e^{iqx(\alpha_1+u\alpha_3)} \tag{4.7}$$

and

$$
\begin{aligned}
&< \pi(q)|\bar{d}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(ux)u(0)|0 >= \\
&if_\pi \Big[q_\beta\Big(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\Big) - q_\alpha\Big(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\Big)\Big] \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)} \\
&+ if_\pi \frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)} \ .
\end{aligned}
\tag{4.8}
$$

In Eq. (4.8) the field $\tilde{G}_{\alpha\beta}$ is the dual of $G_{\alpha\beta}$: $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\delta\rho}G^{\delta\rho}$; the integration on the variables α_i is performed considering that $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$.

Using the normalization of the function φ_P: $\int_0^1 \varphi_P(u)du = 1$ we recover from Eq. (4.2), in the soft pion limit $q \to 0$, the leading order expansion of $B(q_1^2, q_1^2, 0)$ in Eq. (3.13), apart from an overall factor $-2m_b$ due to the the different choice of the current interpolating the B^{**} in the two cases (vector and scalar, respectively); it should be noticed that the terms proportional to the mixed quark-gluon condensate appearing in Eq. (3.13) are missing here since they are related to higher-twist pion wave functions.

The hadronic representation of $A(q_1^2, q_2^2, q^2)$ Eq. (4.1) can be expressed in terms of the contribution of the lowest-lying resonances B and B^{**}:

$$
A_{pole}(q_1^2, q_2^2, q^2) = G_{B^{**}B\pi} f_B f_{B^{**}} \frac{m_B^2 m_{B^{**}}^2}{(m_b+m_d)(m_b-m_u)} \frac{1}{(m_{B^{**}}^2 - q_1^2)(m_B^2 - q_2^2)} \tag{4.9}
$$

in the region $m_b^2 \le s \le s_0$, $m_b^2 \le s' \le s_0'$, and of the contribution of higher states and of the hadronic continuum. For $q_1 \neq q_2$ we can perform a double independent borelization in the variables $-q_1^2$ and $-q_2^2$. In this way, the parasitic contributions coming from the resonance-continuum terms in (4.1) are exponentially suppressed. From Eq. (4.1) we get:

$$
\mathcal{B}\ A_{had} = \frac{1}{M_1^2 M_2^2}\frac{1}{\pi^2}\int ds\ ds' \rho_{had}(s,s',q^2)\exp(-\frac{s}{M_1^2} - \frac{s'}{M_2^2}) \tag{4.10}
$$

where M_1^2 is the Borel parameter associated to the variable $-q_1^2$ and M_2^2 to $-q_2^2$. In this expression the contribution of the pole reads:

$$
\mathcal{B}\ A_{pole} = \frac{1}{M_1^2 M_2^2} G_{B^{**}B\pi} f_B f_{B^{**}} \frac{m_B^2 m_{B^{**}}^2}{(m_b+m_d)(m_b-m_u)} \exp\Big[-\frac{m_{B^{**}}^2}{M_1^2} - \frac{m_B^2}{M_2^2}\Big] \ . \tag{4.11}
$$

On the other hand, borelization of A^{QCD} provides us with the following expression:

$$
\mathcal{B}\ A^{QCD} = \frac{1}{M_1^2 M_2^2}\exp\Big[-\frac{m_b^2}{M^2}\Big]\Big\{ -2\frac{m_b < \bar{u}u >}{f_\pi} M^2 \varphi_P(u_0)
$$

$$
\begin{aligned}
&- f_\pi \frac{M^4}{2}\varphi_\pi'(u_0) + 2f_\pi(M^2+m_b^2)g_1'(u_0) \\
&+ f_\pi \frac{M^2}{2}\Big[\int_0^{u_0}\frac{d\alpha_3}{\alpha_3}[\tilde\chi(u_0-\alpha_3,1-u_0,\alpha_3)-\chi(u_0-\alpha_3,1-u_0,\alpha_3)] \\
&- \int_0^1 \frac{d\alpha_3}{\alpha_3}(\chi(u_0,1-u_0-\alpha_3,\alpha_3)+\tilde\chi(u_0,1-u_0-\alpha_3,\alpha_3)) \\
&+ 2\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3^2}\chi(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\Big]\Big\}\ ,
\end{aligned}
\tag{4.12}
$$

where $M^2 = \frac{M_1^2 M_2^2}{M_2^2+M_2^2}$ and $u_0 = \frac{M_1^2}{M_2^2+M_2^2}$, $\varphi' = \frac{d\varphi}{du}$ and $g_1' = \frac{dg_1}{du}$. It is worth observing that, choosing the symmetric point $u_0 = \frac{1}{2}$ (which corresponds to a quark and an antiquark of the same momentum inside the pion) Eq. (4.12) is considerably simple, since at this point both φ' and g_1' vanish. Moreover, at $u_0 = \frac{1}{2}$ the subtraction of the continuum contribution can be done by substituting $e^{-\frac{m_b^2}{M^2}} \to e^{-\frac{m_b^2}{M^2}} - e^{-\frac{s_0}{M^2}}$, at least in the twist 3 contribution [18] (we use this substitution everywhere in Eq. (4.12) since higher twist contributions are numerically small). Therefore, we derive the sum rule:

$$
\begin{aligned}
&G_{B^{**}B\pi} f_B f_{B^{**}} \frac{m_B^2 m_{B^{**}}^2}{(m_b+m_d)(m_b-m_u)} \exp\Big[-\frac{m_{B^{**}}^2+m_B^2}{2M^2}\Big] = \\
&= \Big\{\exp\Big[-\frac{m_b^2}{M^2}\Big] - \exp\Big[-\frac{s_0}{M^2}\Big]\Big\}\frac{M^2 f_\pi}{2}\Big\{-\frac{4m_b < \bar u u >}{f_\pi^2}\varphi_P(u_0) \\
&+ \int_0^{u_0}\frac{d\alpha_3}{\alpha_3}[\tilde\chi(u_0-\alpha_3,1-u_0,\alpha_3)-\chi(u_0-\alpha_3,1-u_0,\alpha_3)] \\
&- \int_0^1 \frac{d\alpha_3}{\alpha_3}(\chi(u_0,1-u_0-\alpha_3,\alpha_3))+\tilde\chi(u_0,1-u_0-\alpha_3,\alpha_3))) \\
&+ 2\int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3^2}\chi(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)\Big\}\ .
\end{aligned}
\tag{4.13}
$$

Let us now discuss the numerical analysis of Eq.(4.13). In the b-channel we choose the same values for m_b, $m_{B^{**}}$ and $< \bar u u >$ used in Section III. The main nonperturbative quantities are the twist 3 function φ_P and the combinations χ and $\tilde\chi$ of twist 4 wave-functions. We choose the model in Ref. [27], where such quantities have been fixed in the framework of a systematic expansion in the conformal spin. It turns out that, using $\varphi_P(1/2)|_{\mu_b} = 1.07$ [27], the higher twist contribution in Eq.(4.13) is 2% of the twist 3 contribution, and therefore the numerical value of $\varphi_P(1/2)$ represents a crucial quantity in our analysis. We allow the effective threshold s_0 to vary in the range $36 - 40\ GeV^2$. Moreover, we fix the highest

value of the Borel parameter M^2 in the duality window by imposing that the contribution of the continuum is 30% of the resonance. In this way we find $M^2_{max} = 12 - 14\ GeV^2$. The minimum value of M^2 is usually fixed by imposing that terms proportional to higher powers of $1/M^2$ are small enough. Since in Eq.(4.13) such terms are absent, we only look for a stability region in M^2, and choose $M^2_{min} = 6\ GeV^2$, which is the same value adopted in the analysis of $g_{B^* B\pi}$. With these input parameters we find:

$$G_{B^{**} B\pi} f_B f_{B^{**}} = 0.69 \pm 0.14\ GeV^3 \tag{4.14}$$

where the uncertainty is due to the variation of s_0 and to the dependence of the numerical results on M^2. Therefore, using the same values of the leptonic constants f_B and $f_{B^{**}}$ adopted in the previous Section, we get: $G_{B^{**} B\pi} = 21 \pm 7\ GeV$ and, from (3.2), $h(m_b) = -0.52 \pm 0.18$.

In the case of the charm sector, we use $\varphi_P(1/2)|_{\mu_c} = 1.14$, $s_0 = 9 - 11\ GeV^2$ and M^2 in the range $M^2 = 2 - 5\ GeV^2$. We obtain:

$$G_{D^{**} D\pi} f_D f_{D^{**}} = 0.21 \pm 0.02\ GeV^3 \tag{4.15}$$

and the results: $G_{D^{**} D\pi} = 6.3 \pm 1.2\ GeV$ (using f_D and $f_{D^{**}}$ as in Section III) and $h(m_c) = -0.44 \pm 0.09$.

A two-parameter fit of the above results in the form

$$h(m) = h\ (1 + \frac{\sigma}{m}) \tag{4.16}$$

gives the result:

$$h = -0.56 \pm 0.28 \tag{4.17}$$

and for the parameter σ, $\sigma = 0.4 \pm 0.8\ GeV$.

Let us now compare these results with those of the previous Section. The values of h found by the two methods agree with each other. As for the finite mass results, the two methods sensibly differ (almost a factor of 2) in the case of the charm, while the deviation is

less important for the case of beauty (around 40%). These differences should be attributed to corrections to the soft pion limit that have not been incorporated in the results of Section III.

V. EXCITED HEAVY MESONS WIDTHS

In this Section we apply our results for the strong coupling constants to the calculation of the hadronic widths of the excited mesons. First of all, we can compute the strong widths of the decays $P_0 \to P\pi$ and $P_1 \to P^*\pi$. Differently from the decays of the positive parity states having $s_\ell^+ = \frac{3}{2}^+$, i.e. the states P_2 and P_1', where the final pion is in D-wave, these are S-wave decays; moreover, as shown in [11], the single pion channels are expected to saturate the total widths.

In the $m_Q \to \infty$ limit one would obtain

$$\Gamma(P_0 \to P^+\pi^-) = \Gamma(P_1 \to P^{*+}\pi^-) = \frac{1}{2\pi}\left(\frac{h}{f_\pi}\right)^2 \Delta^3 \tag{5.1}$$

but this formula is of limited significance, especially for the case of charm, due to the large $1/m_Q$ corrections coming from the kinematical factors.

Keeping m_Q finite, the formulae become:

$$\Gamma(P_0 \to P^+\pi^-) = \frac{1}{8\pi} G^2_{P^{**}P\pi} \frac{\left[(m^2_{P_0} - (m_P + m_\pi)^2)(m^2_{P_0} - (m_P - m_\pi)^2)\right]^{\frac{1}{2}}}{2m^3_{P_0}} \ . \tag{5.2}$$

Using $G_{D^*D\pi} = 6.3 \pm 1.2\ GeV$, $G_{B^*B\pi} = 21 \pm 7\ GeV$, and $\Delta_D = \Delta_B = 500\ MeV$, one finds

$$\Gamma(D_0 \to D\pi) \simeq 180\ MeV \tag{5.3}$$

$$\Gamma(B_0 \to B\pi) \simeq 360\ MeV\ . \tag{5.4}$$

For the decay $1^+ \to 1^-\pi$ $(P_1 \to P^*\pi)$ we find

$$\begin{aligned}\Gamma(P_1 \to P^{*+}\pi^-) = \frac{G^2_{P_1P^*\pi}}{8\pi} & \frac{\left[(m^2_{P_1} - (m_{P^*} + m_\pi)^2)(m^2_{P_1} - (m_{P^*} - m_\pi)^2)\right]^{\frac{1}{2}}}{2m^3_{P_1}} \times \\ & \times \frac{1}{3}\left(2 + \frac{(m^2_{P_1} + m^2_{P^*})^2}{4m^2_{P_1} m^2_{P^*}}\right) \ . \end{aligned} \tag{5.5}$$

In the limit $m_Q \to \infty$ Eqs.(5.5) and (5.2) coincide. Notice that we have not computed the coupling $G_{P_1 P^* \pi}$. In the infinite-mass limit it coincides with $G_{P^{**} P \pi}$ and therefore, in order to estimate the widths of the 1^+ states, we assume that this equality holds for finite mass as well. From Eq.(5.5) we obtain:

$$\Gamma(D_1 \to D^* \pi) \simeq 165 \; MeV \tag{5.6}$$

$$\Gamma(B_1 \to B^* \pi) \simeq 360 \; MeV \; . \tag{5.7}$$

Also in this case we have taken $m_{P_1} - m_{P^*} = 500$ MeV ($P = B, D$) as suggested by HQET considerations.

In order to perform some comparison with the experimental data, we write down also the formulae giving the widths of the $s_\ell^+ = \frac{3}{2}^+$ states [11]:

$$\Gamma(P_2^0 \to P^+ \pi^-) = \frac{1}{15\pi} \frac{m_P}{m_{P_2}} \frac{h'^2}{\Lambda_\chi{}^2} \frac{|\vec{p}_\pi|^5}{f_\pi{}^2} \tag{5.8}$$

$$\Gamma(P_2^0 \to P^{*+} \pi^-) = \frac{1}{10\pi} \frac{m_{P^*}}{m_{P_2}} \frac{h'^2}{\Lambda_\chi{}^2} \frac{|\vec{p}_\pi|^5}{f_\pi{}^2} \tag{5.9}$$

$$\Gamma(P_1'^0 \to P^{*+} \pi^-) = \frac{1}{6\pi} \frac{m_{P^*}}{m_{P_2}} \frac{h'^2}{\Lambda_\chi{}^2} \frac{|\vec{p}_\pi|^5}{f_\pi{}^2} \; . \tag{5.10}$$

The strong coupling constant $\frac{h'}{\Lambda_\chi}$ can be estimated from the decay widths of the charmed state $D_2(2460) \to D\pi, D^*\pi$; using $\Gamma_{tot}(D_2) = 21 \pm 5$ MeV [28], and assuming that only two body decays are relevant, one gets $\frac{h'}{\Lambda_\chi} \approx 0.55$ GeV^{-1}. From this result and from Eq. (5.10) one obtains for the state $D_1'^0$ the total width $\Gamma_{tot} \approx 6$ MeV; on the other hand the experimental width of the other narrow state observed in the charm sector, i.e. the 1^+ $D_1(2420)$ particle, is $\Gamma_{tot}(D_1(2420)) = 18 \pm 5$ MeV [28]. This discrepancy could be attributed to a mixing between the D_1 and the D_1' states [29]. If α is the mixing angle, we have

$$\sin^2(\alpha) \approx \frac{12 \; MeV}{\Gamma(D_1) - \Gamma(D_1')} \simeq 0.08 \tag{5.11}$$

and therefore we get the estimate $\alpha \approx 16^o$. This determination agrees with the result of Kilian et al. in Ref. [11].

As for the B sector, evidence has been recently reported of a bunch of positive parity states B^{**}, with an average mass $m_{B^{**}} = 5732 \pm 5 \pm 20$ MeV and an average width $\Gamma(B^{**}) =$

145 ± 28 MeV [30,31]. The observed states can be identified with the two doublets $(2^+, 1^+)$ and $(1^+, 0^+)$. We note that the mass splitting, $\Delta = 500\ MeV$, between S and P states that we have chosen, agrees rather well with the experimental result in the B sector; our predictions for the widths, using the experimental value for the B^{**} mass, are: $\Gamma_{tot}(B_0) \simeq 330\ MeV$, $\Gamma_{tot}(B_1) \simeq 300\ MeV$, $\Gamma_{tot}(B_2) \simeq 12$ MeV and $\Gamma_{tot}(B_1^\prime) \simeq 10$ MeV (we have neglected here the mixing which is a $1/m_Q$ effect).

It is difficult to perform a detailed comparison of these results with the yet uncomplete experimental outcome; however, assuming that the result obtained by LEP collaborations represents an average of several states, its value is compatible with our estimate of the widths. Opal [31] has also reported evidence of a B_s^{**} state with mass $m_{B_s^{**}} = 5853 \pm 15$ MeV and width $\Gamma_{B_s^{**}} = 47 \pm 22$ MeV. The width can be interpreted as connected to the decay $B_s^{**} \to BK,\ B^* K$. Assuming again that the width is saturated by two-particle final states, and using $m_{B_s^{**}} = 5853\ MeV$, we obtain:

$$\Gamma(B_s^{**}(0^+)) \simeq 280\ MeV \tag{5.12}$$

$$\Gamma(B_s^{**} \to B^* K) \simeq 200\ MeV \qquad (s_\ell = \frac{1}{2}^+) \tag{5.13}$$

$$\Gamma(B_s^{**}(1^+)) \simeq 0.45\ MeV \qquad (s_\ell = \frac{3}{2}^+) \tag{5.14}$$

$$\Gamma(B_s^{**}(2^+)) \simeq 1.4\ MeV\ . \tag{5.15}$$

Also in this case a detailed comparison with the experimental results cannot be performed without more precise measurements; we observe, however, that the computed widths of the different B_s^{**} states are generally smaller than the corresponding quantities of the B^{**} particles, a feature which is reproduced by the experiment.

VI. EXCITED STATES CONTRIBUTION TO f_{D_S}/f_D

Aim of this section is to study the contribution of the excited heavy mesons to the ratio of leptonic decay constants f_{D_s}/f_D. They are defined as

$$< 0|\bar{d}\gamma_\mu\gamma_5 c|D^+(p) >= i f_D p_\mu \tag{6.1}$$

$$< 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(p) >= i f_{D_s} p_\mu \tag{6.2}$$

In the chiral $SU(3)$ limit such a ratio is one, and the chiral corrections are expected to be of the order m_s/Λ_χ. In terms of the fields H_a and ξ defined in Section II, the left-handed current $\bar{q}_a\gamma_\mu(1-\gamma_5)Q$ corresponds to [3]

$$L^a_\mu = \frac{i}{2}\hat{F} < \gamma_\mu(1-\gamma_5)H_b\xi^\dagger_{ba} > + \dots \tag{6.3}$$

where the dots denote terms of higher order in the heavy mass and chiral expansion. At the lowest order, $f_{D_s} = f_D = \hat{F}/\sqrt{m_D}$. The effective couplings of the higher order terms, contributing to $SU(3)$-violating corrections to the ratio f_{D_s}/f_D, are unknown. In addition, there are non-analytic corrections arising from chiral loops: in previous works [33,34] the one-loop "log-enhanced" terms of the form $m^2\log(m^2/\mu^2)$ ($m = m_\pi$, m_K or m_η) were kept, giving

$$\frac{f_{D_s}}{f_D} = 1 + 0.07 + 0.21g^2 \tag{6.4}$$

The effective coupling g, appearing in the heavy-light chiral lagrangian (2.5), gives the vertex $D^*D\pi$, appearing in the loops.

The loop corrections depend on an arbitrary renormalization point μ: this dependence is canceled by the μ-dependence of the coefficients of higher order operators, which are here neglected. When μ is of the order of the chiral symmetry breaking scale $\Lambda_\chi \approx 1\ GeV$, these higher terms do not contain large logarithms and are supposed to be small compared with the ones coming from the chiral loops.

The excited positive parity heavy mesons contribute to $SU(3)$ violating effects as virtual intermediate states in chiral loops. In Ref. [12] the "log-enhanced" terms due to these excited-states loops has been computed: as we will see below, some of them are proportional to h^2 and others depend linearly on h. It has been pointed out that these terms could be numerically relevant and could invalidate the chiral estimate based only on the states D and D^*; as we shall see below, however, the terms $\mathcal{O}(h^2)$ and $\mathcal{O}(h)$, while important, tend to cancel.

In the following, we will present an independent calculation of the chiral loop contributions to the ratio of leptonic decay constants, and we will give a numerical estimate obtained by using the QCD sum rules results for the couplings g and h.

The chiral loop induced corrections to f_D and f_{D_s} come from the diagrams of figs. 2,3 and 4.

The self-energy diagrams (fig. 2) give the following wave function renormalization factors:

$$
\begin{aligned}
Z_D = 1 &- \frac{3g_D^2}{16\pi^2 f_\pi^2}\left[3/2C_1(\Delta_{D^*D},\Delta_{D^*D},m_\pi)+C_1(\Delta_{D_s^*D},\Delta_{D_s^*D},m_K)+\frac{1}{6}C_1(\Delta_{D^*D},\Delta_{D^*D},m_\eta)\right]\\
&+ \frac{h_D^2}{16\pi^2 f_\pi^2}\left[3/2C(\Delta_{P_0P},\Delta_{P_0P},m_\pi)+C(\Delta_{P_{0s}P},\Delta_{P_{0s}P},m_K)+\right.\\
&+ \left.\frac{1}{6}C(\Delta_{P_0P},\Delta_{P_0P},m_\eta)\right]
\end{aligned}
\tag{6.5}
$$

$$
\begin{aligned}
Z_{D_s} = 1 &- \frac{3g_D^2}{16\pi^2 f_\pi^2}\left[2C_1(\Delta_{D^*D_s},\Delta_{D^*D_s},m_K)+\frac{2}{3}C_1(\Delta_{D^*D},\Delta_{D^*D},m_\eta)\right]\\
&+ \frac{h_D^2}{16\pi^2 f_\pi^2}\left[2C(\Delta_{P_0P_s},\Delta_{P_0P_s},m_K)+\frac{2}{3}C(\Delta_{P_0P},\Delta_{P_0P},m_\eta)\right]
\end{aligned}
\tag{6.6}
$$

where the mass splittings $\Delta_{P^*P}=M_{P^*}-M_P$, $\Delta_{P^*P_s}=M_{P^*}-M_{P_s}$ and $\Delta_{P_s^*P}=M_{P_s^*}-M_P$ are $\mathcal{O}(1/m_Q)$, while the mass splittings $\Delta_{P_0P}=M_{P_0}-M_P$, $\Delta_{P_{0s}P}=M_{P_{0s}}-M_P$ and $\Delta_{P_0P_s}=M_{P_0}-M_{P_s}$ between excited and ground states are finite in the limit $m_Q\to\infty$.

The functions C_1 and C come from the loop integration and are defined as:

$$
\int\frac{d^4}{(2\pi)^4}\frac{q_\alpha q_\beta}{(q^2-m^2)(v\cdot q-\Delta)(v\cdot q-\Delta')}=\frac{i}{16\pi^2}\left(C_1(\Delta,\Delta',m)g_{\alpha\beta}+C_2(\Delta,\Delta',m)v_\alpha v_\beta\right)
\tag{6.7}
$$

and

$$
C(\Delta,\Delta',m)=C_1(\Delta,\Delta',m)+C_2(\Delta,\Delta',m)\ .
\tag{6.8}
$$

Performing the integration we obtain:

$$
C_1(\Delta,\Delta',m)=\frac{m^3}{9(\Delta-\Delta')}\left[H_1(\frac{\Delta}{m},m)-H_1(\frac{\Delta'}{m},m)\right]
\tag{6.9}
$$

$$
C_(\Delta,\Delta',m)=\frac{2m^3}{9(\Delta-\Delta')}\left[H(\frac{\Delta}{m},m)-H(\frac{\Delta'}{m},m)\right]
\tag{6.10}
$$

where

$$H_1(x,m) = -12x + 10x^3 + (9x - 6x^3)\log(\frac{m^2}{\mu^2}) + \\ - 12(x^2-1)^{3/2}\log(x+\sqrt{x^2-1}) \tag{6.11}$$

$$H(x,m) = -9x^3 + (9x^3 - \frac{9}{2}x)\log(\frac{m^2}{\mu^2}) + \\ + 18x^2\sqrt{x^2-1}\log(x+\sqrt{x^2-1}) \ . \tag{6.12}$$

The previous results are obtained in a renormalization scheme such that $\frac{2}{\epsilon}+\log(4\pi)-\gamma_E+1 = 0$.

For $\Delta = \Delta'$, as for the wave-function renormalization factors, we find

$$C_1(\Delta,\Delta,m) = \frac{m^2}{9}H_1'(\frac{\Delta}{m},m) \tag{6.13}$$

$$C(\Delta,\Delta,m) = \frac{2m^2}{9}H'(\frac{\Delta}{m},m) \tag{6.14}$$

where $H'(x,m) = \frac{dH(x,m)}{dx}$.

The diagram of fig. 3 is linear in h (the analogous proportional to g vanishes), and proportional to $\hat{F}^+$, defined in (3.29).

Combining all the diagrams (fig. 2, 3 and 4) we obtain:

$$\begin{aligned} f_D = \frac{\hat{F}}{\sqrt{M_D}}\Bigg[1 &- \frac{1}{32\pi^2 f_\pi^2}\left[\frac{3}{2}m_\pi^2\log(\frac{m_\pi^2}{\mu^2}) + m_K^2\log(\frac{m_K^2}{\mu^2}) + \frac{1}{6}m_\eta^2\log(\frac{m_\eta^2}{\mu^2})\right] \\ &- \frac{3g_D^2}{32\pi^2 f_\pi^2}\left[\frac{3}{2}C_1(\Delta_{D^*D},\Delta_{D^*D},m_\pi) + C_1(\Delta_{D_s^*D},\Delta_{D_s^*D},m_K) + \frac{1}{6}C_1(\Delta_{D^*D},\Delta_{D^*D},m_\eta)\right] \\ &+ \frac{h_D^2}{32\pi^2 f_\pi^2}\left[\frac{3}{2}C(\Delta_{P_0P},\Delta_{P_0P},m_\pi) + C(\Delta_{P_{0s}P},\Delta_{P_{0s}P},m_K) + \frac{1}{6}C(\Delta_{P_0P},\Delta_{P_0P},m_\eta)\right] \\ &+ \frac{\hat{F}^+}{\hat{F}}\frac{h_D}{16\pi^2 f_\pi^2}\left[\frac{3}{2}C(\Delta_{P_0P},0,m_\pi) + C(\Delta_{P_{0s}P},0,m_K) + \frac{1}{6}C(\Delta_{P_0P},0,m_\eta)\right]\Bigg] \end{aligned} \tag{6.15}$$

$$\begin{aligned} f_{D_s} = \frac{\hat{F}}{\sqrt{M_D}}\Bigg[1 &- \frac{1}{32\pi^2 f_\pi^2}\left[2m_K^2\log(\frac{m_K^2}{\mu^2}) + \frac{2}{3}m_\eta^2\log(\frac{m_\eta^2}{\mu^2})\right] \\ &- \frac{3g_D^2}{32\pi^2 f_\pi^2}\left[2C_1(\Delta_{D^*D_s},\Delta_{D^*D_s},m_K) + \frac{2}{3}C_1(\Delta_{D^*D},\Delta_{D^*D},m_\eta)\right] \\ &+ \frac{h_D^2}{32\pi^2 f_\pi^2}\left[2C(\Delta_{P_0P_s},\Delta_{P_0P_s},m_K) + \frac{2}{3}C(\Delta_{P_0P},\Delta_{P_0P},m_\eta)\right] \\ &+ \frac{\hat{F}^+}{\hat{F}}\frac{h_D}{16\pi^2 f_\pi^2}\left[2C(\Delta_{P_0P_s},0,m_K) + \frac{2}{3}C(\Delta_{P_0P},0,m_\eta)\right]\Bigg] \ . \end{aligned} \tag{6.16}$$

From the previous formulae, using $\Delta_{P_0P} = 0.5\ GeV$, $\mu = 1$, $\hat{F}^+ = 0.46\ GeV^{3/2}$ and $\hat{F} = 0.30\ GeV^{3/2}$, one gets numerically:

$$f_D = \frac{\hat{F}}{\sqrt{M_D}} \Big(1 + 0.09 + 0.11 g^2 - 0.33 h^2 - 1.00 h\Big) \tag{6.17}$$

$$f_{D_s} = \frac{\hat{F}}{\sqrt{M_D}} \Big(1 + 0.17 + 0.82 g^2 - 0.66 h^2 - 1.15 h\Big)\ . \tag{6.18}$$

In the previous formulae we have kept only the leading order in the $1/m_Q$, i.e. we have put $\Delta_{D^*D} = 0$ in (6.15) and (6.16): therefore we have to use for the couplings g_D and h_D the asymptotic values g and h respectively.

The value of g has been computed with QCD sum rules in [17,18,32], giving a result in the range $0.2 - 0.4$. From (6.17) and (6.18), using $g = 0.3$ and $h = -0.5$, we get for the ratio of leptonic decay constants:

$$\frac{f_{D_s}}{f_D} \simeq 1.09\ . \tag{6.19}$$

Without the contibution of the excited states, we would get $f_{D_s}/f_D \simeq 1.13$: the contribution of the excited heavy mesons is slightly negative. Notice that the term in h^2 tends to cancel against the term linear in h; we also observe that its sign is unambiguously fixed by the sum rule (see Eq.(3.30), since the relevant quantity is the ratio $h\hat{F}^+/\hat{F}$.

The result (6.19) is not very sensitive to the value of the mass splitting Δ_{D_0D}. For instance, if we take $\Delta_{D_0D} = 0.6\ GeV$ we find $f_{D_s}/f_D = 1.12$, while for $\Delta_{D_0D} = 0.4\ GeV$ one obtains $f_{D_s}/f_D = 1.05$. We have also checked that (6.19) depends weakly on the value of the renormalization scale μ.

Keeping only the "log-enhanced" terms of the form $m^2 \log(m^2/\mu^2)$, the ratio becomes:

$$\frac{f_{D_s}}{f_D} = 1 - \frac{1}{32\pi^2 f_\pi^2}\left[m_K^2 \log(\frac{m_K^2}{\mu^2}) + \frac{1}{2} m_\eta^2 \log(\frac{m_\eta^2}{\mu^2}) - \frac{3}{2} m_\pi^2 \log(\frac{m_\pi^2}{\mu^2})\right] \times$$
$$\times \left(1 + 3g^2 + h^2 + \frac{\hat{F}^+}{\hat{F}} h\right) \simeq 1.06 \tag{6.20}$$

The previous formula does not contain the parameter Δ_{D_0D}. The quoted number, 1.06, is for $\mu = 1\ GeV$: putting $h = 0$ in (6.20), one obtains the results of [33,34], i.e. Eq.(6.4). A

different approximation has been put forward in [12], where also terms $\Delta^2 \log(\Delta^2/\mu^2)$ are kept, with numerical results similar to ours.

In conclusion, the contribution of the excited heavy meson states to the ratio of leptonic decay constants f_{D_s}/f_D is small and negative. The usual estimate of the $SU(3)$ violation, including only the state D and D^*, is not destabilized when including excited states, at least in this case. We stress that this result strongly depends on the sign of h: had it been positive, the final result would have been substantially different. Therefore we cannot exclude that for other observables the positive parity heavy mesons give a significant contribution to the chiral loops. In any case, we point out that the sign of the product $\hat{F}^+/\hat{F}h$, which enters in the formula (6.20), is unambiguously determined by the sum rule. We also observe that the sign of h turns out to be negative also in ref. [35], based on a chiral quark model.

The numerical outcome Eq.(6.20) for f_{D_s}/f_D agrees with the theoretical results obtained by several groups by different models, e.g. QCD sum rules [36], lattice QCD [37] and potential models [38]. As for the experimental results, we only have the upper bound [28]

$$f_D \leq 310 \; MeV \tag{6.21}$$

from MARK III collaboration [39], and the recent results from three experiments:

$$\begin{aligned}
f_{D_s} &= 232 \pm 45 \pm 20 \pm 48 \; MeV \quad (WA75 \; [40]) \\
f_{D_s} &= 344 \pm 37 \pm 52 \pm 42 \; MeV \quad (CLEO \; [41]) \\
f_{D_s} &= 430^{+150}_{-130} \pm 40 \; MeV \qquad\quad (BES \; [42]) \; .
\end{aligned} \tag{6.22}$$

Even though the theoretical value obtained for f_{D_s}/f_D is still consistent with data in (6.21),(6.22), it should be noticed that the experimental results for f_{D_s} seem to indicate a value much larger than the theoretical estimates appeared in the literature [36–38], which might signal a serious theoretical problem. In any event, better quality data are needed before any conclusion can be drawn.

VII. CONCLUSIONS

We have computed the strong coupling constant of the positive parity heavy mesons $G_{B^{**}B\pi}$ and $G_{D^{**}D\pi}$, by QCD sum rules and light-cone sum rules. In the limit $m_Q \to \infty$, the two methods give compatible results, but the $1/m_Q$ corrections are significant and unfortunately they are rather different in the two approaches. We have applied the results to the calculation of the hadronic widths of the positive parity B and D states: we have found that the calculated widths are in any case compatible with the recent preliminary LEP data on the orbitally excited B mesons. Furthermore, we have computed the chiral loop contributions of these states to the ratio f_{D_s}/f_D, and we have found that the chiral corrections consist of two sizeable quantities, which are however opposite in sign, so that the prediction for this ratio obtained using only the ground state heavy mesons is not significantly shifted. This cancellation is likely here to be fortuitous, and in view of the large value we have found for the coupling constant, leaves open the possibility that chiral contributions of the excited heavy mesons to other physical observables could instead be important.

ACKNOWLEDGMENTS

We thank F. Feruglio, M. Lenti, A.Palano and Zhao-Xi Zhang for useful discussions.

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FIGURES

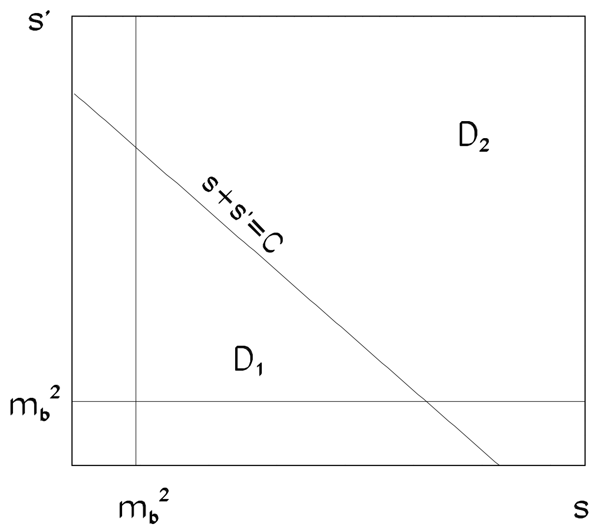

FIG. 1. The integration region of the double dispersive integral in Eq. (3.11). In the region D_1 only the poles of the B, B^* and B^{**} particles are present, the region D_2 includes the hadronic continuum.

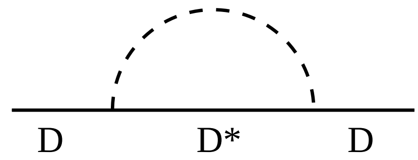

FIG. 2. Self energy diagram

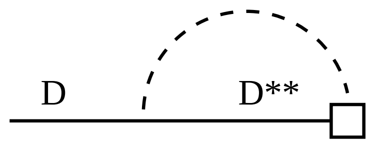

FIG. 3. Vertex correction involving positive parity heavy mesons

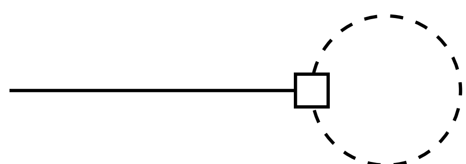

FIG. 4. Vertex correction involving only light pseudoscalar mesons